SI Session: September 2nd & 4th, 2008
Mondays: 3:00 PM – 4:30 PM
Tuesdays: 1:30 PM – 3:00 PM
Thursdays: 1:30 PM – 3:00 PM
Room 1239 SNAD

Prof. McCurdy : Linear Algebra
Fall 2008
SI Leader : Neil Jody

[1] Find the general solutions of the systems whose augmented matrices are given.

(a) $\begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ -1 & 7 & -4 & 2 & 7 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & -5 & -6 & 0 & -5 \\ 0 & 1 & -6 & -3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

[2] Suppose each matrix represents the augmented matrix for a system of linear equations. In each case, determine if the system is consistent. If the system is consistent, determine if the solution is unique. The leading entries $(\bullet)$ may have any nonzero value; the starred entries $(*)$ may have any values (including zero).

(a) $\begin{bmatrix} \bullet & * & * \\ 0 & \bullet & * \\ 0 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} \bullet & * & * & * & * \\ 0 & 0 & \bullet & * & * \\ 0 & 0 & 0 & \bullet & * \end{bmatrix}$

(c) $\begin{bmatrix} \bullet & * & * & * & * & * \\ 0 & \bullet & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & \bullet \\ 0 & 0 & \bullet & * & * & * \end{bmatrix}$

(d) $\begin{bmatrix} \bullet & * & * & * \\ 0 & \bullet & * & * \\ 0 & 0 & \bullet & * \\ 0 & 0 & 0 & 0 \end{bmatrix}$

[3] Determine the value(s) of h such that the matrix is the augmented matrix of a consistent linear system.

$$\begin{bmatrix} 1 & -3 & -2 \\ 5 & h & -7 \end{bmatrix}$$

[4] Choose h and k such that the system has (a) no solution, (b) a unique solution, and (c) many solutions. Give separate answers for each part.

$$\begin{aligned} x_1 + 3x_2 &= 2 \\ 3x_1 + hx_2 &= k \end{aligned}$$

[5] Suppose a system of linear equations has a 3×5 augmented matrix whose fifth column is a pivot column. Is the system consistent? Why or why not?

[6] Suppose the coefficient matrix of a linear system of three equations in three variables has a pivot in each column. Explain why the system has a unique solution.

[7] A system of linear equations with fewer equations than unknowns is sometimes called an *underdetermined system*. Give an example of an underdetermined system of two equations in three unknowns.

[8] Determine if $\vec{b}$ is a linear combination of $\vec{a}_1$, $\vec{a}_2$, and $\vec{a}_3$.

(a) $\vec{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$, $\vec{a}_2 = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}$, $\vec{a}_3 = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$

(b) $\vec{a}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\vec{a}_2 = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}$, $\vec{a}_3 = \begin{bmatrix} -6 \\ 7 \\ 5 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$

[9] Let $\vec{v}_1 = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}$.List five vectors in $\text{Span}\{\vec{v}_1, \vec{v}_2\}$. For each vector, show the weights on $\vec{v}_1$ and $\vec{v}_2$ used to generate the vector and list the three entries of the vector. Give a geometric description of $\text{Span}\{\vec{v}_1, \vec{v}_2\}$.

[10] Let $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} -3 \\ 1 \\ 8 \end{bmatrix}$, $\vec{y} = \begin{bmatrix} h \\ -5 \\ -3 \end{bmatrix}$. For what value(s) of h is $\vec{y}$ in the plane generated by $\vec{v}_1$ and $\vec{v}_2$?

[11] Let $A = \begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix}$, let $\vec{b} = \begin{bmatrix} 10 \\ 3 \\ 3 \end{bmatrix}$, and let W be the set of all linear combinations of the columns of A.

(a) Is $\vec{b}$ in W?

(b) Show that the third column of A is in W.

[12] Given $A = \begin{bmatrix} 1 & 2 & 1 \\ -3 & -1 & 2 \\ 0 & 5 & 3 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$, write the augmented matrix for the linear system that corresponds to the matrix equation $A\vec{x} = \vec{b}$. Then solve the system and write the solution as a vector.

[13] Let $\vec{u} = \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}$ and $A = \begin{bmatrix} 5 & 8 & 7 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{bmatrix}$. Is $\vec{u}$ in the subset of $\mathbb{R}^3$ spanned by the columns of A? Why or Why not?