SI Session: October 20th,21st,23rd 2008
Mondays: 3:00 PM – 4:30 PM
Tuesdays: 1:30 PM – 3:00 PM
Thursdays: 1:30 PM – 3:00 PM
Room 1239 SNAD

Prof. McCurdy : Linear Algebra
Fall 2008
SI Leader : Neil Jody

[1] Let $V = R^2$ with the usual scalar multiplication and with vector addition defined by $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 + y_2 \\ x_2 + y_1 \end{bmatrix}$. Determine if V under these operations satisfies the following axioms:

(a) $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$ for all $\vec{u}, \vec{v}, \vec{w}$ in V
(b) $s(\vec{u} + \vec{v}) = s\vec{u} + s\vec{v}$ for all $\vec{u}, \vec{v}$ in V and s in R
(c) $(s + t)\vec{v} = s\vec{v} + t\vec{v}$ for all s, t in R and v in V

[2] Determine if each set is a subspace of R^3 under its usual operations.

(a) $W = \left\{ \begin{bmatrix} a \\ a \\ a-1 \end{bmatrix} : a \in R \right\}$

(b) $W = \left\{ \begin{bmatrix} a \\ a+b \\ b \end{bmatrix} : a, b \in R \right\}$

[3] Let V be a vector space of dimension 4. Determine if each statement is true or false.

(a) Any set of 5 vectors in V must be linearly dependent.
(b) Any set of 5 vectors in V must span V
(c) Any set of 3 vectors in V must be linearly independent.
(d) No set of 3 vectors in V can span V.

[4] Let W be the set of all vectors of the form shown, where a, b, and c represent arbitrary real numbers. In each case, either find a set S of vectors that spans W or give an example to show that W is *not* a vector space.

(a) $\begin{bmatrix} -a+1 \\ a-6b \\ 2b+a \end{bmatrix}$

(b) $\begin{bmatrix} 4a+b \\ 0 \\ a+b+c \\ c-2a \end{bmatrix}$

[5] A matrix A is said to be an orthogonal matrix if $AA^T = I$. Find a 2×2 matrix (other than the identity matrix) that is orthogonal.

[6] Find an explicit description of Nul A, by listing vectors that span the null space.

(a) $A = \begin{bmatrix} 1 & -6 & 4 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 5 & -4 & -3 & 1 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

[7] Either use an appropriate theorem to show that a given set , W , is a vector space, or find a specific example to the contrary.

(a) $\left\{ \begin{bmatrix} r \\ s \\ t \end{bmatrix} : 5r - 1 = s + 2t \right\}$

(b) $\left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} : \begin{matrix} a + 3b = c \\ b + a + c = d \end{matrix} \right\}$

(c) $\left\{ \begin{bmatrix} b-5d \\ 2b \\ 2d+1 \\ d \end{bmatrix} : b, d \text{ real} \right\}$

(d) $\left\{ \begin{bmatrix} -a+2b \\ a-2b \\ 3a-6b \end{bmatrix} : a, b \text{ real} \right\}$

[8] Find A such that the given set is Col A. $\left\{ \begin{bmatrix} b-c \\ 2b+c+d \\ 5c-4d \\ d \end{bmatrix} : b, c, d \text{ real} \right\}$

[9] For the following matrices, (1) find k such that Nul A is a subspace of $\mathbb{R}^k$, and (2) find k such that Col A is a subspace of $\mathbb{R}^k$.

(a) $A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 0 & -5 \\ 0 & -5 & 7 \\ -5 & 7 & -2 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & -3 & 9 & 0 & -5 \end{bmatrix}$

[10] Let $A = \begin{bmatrix} 1 & 3 & 5 & 0 \\ 0 & 1 & 4 & -2 \end{bmatrix}$, find a nonzero vector in Nul A and a nonzero vector in Col A.

[11] Let $A = \begin{bmatrix} -8 & -2 & -9 \\ 6 & 4 & 8 \\ 4 & 0 & 4 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$. Determine if $\vec{w}$ is in Col A.
Is $\vec{w}$ in Nul A?

[12] Determine if the set of polynomials $\{x^2 - 2x + 1, 2x^2 + 3x - 4, -x^2 + x + 5\}$ is a linearly independent set in P_2. Is it a basis for P_2? Why or why not?

[13] Let $S = \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ -4 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} \right\}$, a subset of R^3. Find the dimension of the span of *S*.

[14] Suppose that $\{u, v, w\}$ is a basis for the vector space V. Determine if each of the following sets of vectors is a basis for V. Justify all assertions.

(a) $\{v, u - w, v + 2w, u + v + w\}$
(b) $\{v + w, u + v, -u + v + 2w\}$

[15] Let $A = \begin{bmatrix} 2 & -1 & 3 & 2 \\ 3 & -2 & 1 & -1 \\ -5 & 4 & 3 & 7 \end{bmatrix}$.

(a) Find a basis for the null space of A.
(b) Find a basis for the column space of A.
(c) Find a basis for the row space of A.