SI Session: October 14, 2008
Mondays: 3:00 PM – 4:30 PM
Tuesdays: 1:30 PM – 3:00 PM
Thursdays: 1:30 PM – 3:00 PM
Room 1239 SNAD

Prof. McCurdy : Linear Algebra
Fall 2008
SI Leader : Neil Jody

[1] Determine which of the matrices are invertible. Use as few calculations as possible . Justify your answer.

(a) $\begin{bmatrix} -4 & 6 \\ 6 & -9 \end{bmatrix}$

(b) $\begin{bmatrix} -7 & 0 & -4 \\ 3 & 0 & -1 \\ 2 & 0 & 9 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -5 & -4 \\ 0 & 3 & 4 \\ -3 & 6 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 3 & 7 & 4 \\ 0 & 5 & 9 & 6 \\ 0 & 0 & 2 & 8 \\ 0 & 0 & 0 & 10 \end{bmatrix}$

[2] When is a square lower triangular matrix invertible? Justify your answer.

[3] Is it possible for a 5×5 to be invertible when its columns do not span $\square^5$? Why or Why not?

[4] If C is 6×6 and the equation $C\vec{x} = \vec{v}$ is consistent for every $\vec{v}$ in $\mathbb{R}^6$, is it possible that for some $\vec{v}$, the equation $C\vec{x} = \vec{v}$ has more than one solution? Why or Why not?

[5] The matrices *A* and *B* are said to commute if $AB = BA$. Find all matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ that commute with $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ (Note: there are infinitely many, so specify the *form* of these matrices).

[6] If $n \times n$ matrices E and F have the property that $EF = I$, then E and F commute. Explain why.

[7] If the equation $H\vec{x} = \vec{c}$ is inconsistent for some $\vec{c}$ in $\mathbb{R}^n$, what can you say about the equation $H\vec{x} = \vec{0}$? Why?

[8] If L is $n \times n$ and the equation $L\vec{x} = \vec{0}$ has the trivial solution do the columns of L span $\mathbb{R}^n$? Why?

[9] Explain why the columns of A^2 span whenever the columns of A are linearly independent.

[10] Show that if AB is invertible, so is B.

[11] If A is an $n \times n$ matrix and the transformation $\vec{x} \mapsto A\vec{x}$ is one-to-one, what else can you say about this transformation? Justify your answer.

[12] Suppose A is an $n \times n$ matrix with the property that the equation $A\vec{x} = \vec{0}$ has only the trivial solution. Without using the Invertible Matrix Theorem, explain directly why the equation $A\vec{x} = \vec{b}$ must have a solution for each $\vec{b}$ in $\mathbb{R}^n$.

[13] T is a linear transformation from $\mathbb{R}^2$ into $\mathbb{R}^2$. Show that T is invertible and find a formula for T^{-1}.

$$T(x_1, x_2) = (6x_1 - 8x_2, -5x_1 + 7x_2)$$