SI Session: November 6^{th} &10^{th} 2008
Mondays: 3:00 PM – 4:30 PM
Tuesdays: 1:30 PM – 3:00 PM
Thursdays: 1:30 PM – 3:00 PM
Room 1239 SNAD

Prof. McCurdy : Linear Algebra
Fall 2008
SI Leader : Neil Jody

[1] Compute the determinants of the following matrices by cofactor expansions.

(a) $\begin{bmatrix} 1 & -2 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & -6 & -7 & 5 \\ 5 & 0 & 4 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 4 & 0 & 0 & 0 \\ 7 & -1 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 5 & -8 & 4 & -3 \end{bmatrix}$

(c) $\begin{bmatrix} 6 & 3 & 2 & 4 & 0 \\ 9 & 0 & -4 & 1 & 0 \\ 8 & -5 & 6 & 7 & 1 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 2 & 3 & 2 & 0 \end{bmatrix}$

[2] Find the determinants for the following matrices by row reduction to echelon form.

(a) $$\begin{bmatrix} 1 & 3 & 3 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 5 & 4 & -3 \\ -3 & -7 & -5 & 2 \end{bmatrix}$$

(b) $$\begin{bmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -1 & -6 \\ -2 & -6 & 2 & 3 & 9 \\ 3 & 7 & -3 & 8 & -7 \\ 3 & 5 & 5 & 2 & 7 \end{bmatrix}$$

[3] Combine the methods of row reduction and cofactor expansion to compute the determinants of the following matrices.

(a) $\begin{bmatrix} -1 & 2 & 3 & 0 \\ 3 & 4 & 3 & 0 \\ 5 & 4 & 6 & 6 \\ 4 & 2 & 4 & 3 \end{bmatrix}$

(b) $\begin{bmatrix} -3 & -2 & 1 & -4 \\ 1 & 3 & 0 & -3 \\ -3 & 4 & -2 & 8 \\ 3 & -4 & 0 & 4 \end{bmatrix}$

[4] Find the determinants of the following matrices where $\det\left(\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\right) = 7$.

(a) $\begin{vmatrix} a & b & c \\ 3d & 3e & 3f \\ g & h & i \end{vmatrix}$

(b) $\begin{vmatrix} g & h & i \\ a & b & c \\ d & e & f \end{vmatrix}$

(c) $\begin{vmatrix} a+d & b+e & c+f \\ d & e & f \\ g & h & i \end{vmatrix}$

[5] Show that if A is invertible then $\det A^{-1} = \dfrac{1}{\det A}$.

[6] Let A and B be 4×4 matrices such that $\det A = -2$ and $\det B = 3$. Calculate $\det\left(2A^4B^{-3}\right)$.

[7] Show that if A is invertible and B is not invertible, then AB is not invertible.

[8] Recall that a square matrix A is orthogonal if $AA^T = I$. Show that if A is orthogonal, then $\det A = \pm 1$.