SI Session: Final Exam Review
Saturday, December 6th :
2:00 PM – 4:00 PM
Monday, December 8th:
3:00 PM – 4:30 PM
Room 1239 SNAD

Prof. McCurdy : Linear Algebra
Fall 2008
SI Leader : Neil Jody

[1] Solve the following system. $\begin{cases} x_1 + 3x_2 - 2x_3 = 10 \\ 2x_1 + 3x_2 + 5x_3 = -1 \\ -x_1 + 2x_2 + 4x_3 = -9 \end{cases}$

[2] Express the vector $\begin{bmatrix} 3 \\ -1 \\ -8 \end{bmatrix}$ as a linear combination of the vectors $\begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ 7 \\ 3 \end{bmatrix}$.

[3] Consider the linear transformation $T:\mathbb{R}^3 \to \mathbb{R}^3$ defined by

$$T(\vec{x}) = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 4 & -2 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}. \text{ Let } \vec{u} = \begin{bmatrix} -2 \\ -1 \\ 5 \end{bmatrix}$$

(a) Find the image of T under $\vec{u}$

(b) Is $\vec{u}$ in the range of T?

[4] Suppose that the RREF of a matrix A is $\begin{bmatrix} 1 & 0 & -2 & 0 & 4 \\ 0 & 1 & 3 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

Write the solution set of the equation $A\vec{x} = \vec{0}$ in parametric vector form.

[5] Are the vectors $\begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \\ 2 \\ -1 \end{bmatrix}$, and $\begin{bmatrix} 2 \\ 3 \\ 1 \\ 2 \end{bmatrix}$ *linearly dependent* or *linearly independent*?(Justify your answer.)

[6] Do the vectors $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 1 \\ -8 \end{bmatrix}$ *span* $\mathbb{R}^3$. (Justify your answer.)

[7] Consider the linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^3$ defined by

$$T(\vec{x}) = \begin{bmatrix} 1 & 4 \\ 2 & 3 \\ 1 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

(a) Is T one-to-one?
(b) Is T onto?
(Justify your answer.)

[8] Let $A = \begin{bmatrix} 0 & 2 & 5 \\ 1 & 1 & 2 \\ 2 & 3 & 6 \end{bmatrix}$.

(a) Calculate $2A + A^T$.

(b) Calculate A^{-1}.

[9] Let $T:\mathbb{R}^3 \to \mathbb{R}^2$ be defined by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x-z \\ 2y \end{bmatrix}$. Show that T is a linear transformation.

[10] Let $S=\{(a,b,2a+b) | a,b\in \mathbb{R}\}$. Prove that S is a subspace of $\mathbb{R}^3$.

[11] Let S be a set of all ordered pairs $\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ with vector addition defined by

$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}+\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}=\begin{bmatrix} u_2+v_2 \\ u_1+v_1 \end{bmatrix}$ and scalar multiplication defined by $c\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}=\begin{bmatrix} cu_1 \\ u_2 \end{bmatrix}$.

Determine whether S satisfies Axiom 8 : $(c+d)\vec{u}=c\vec{u}+d\vec{u}$

[12] Decide whether the polynomials $p_1 = x^2 - 2x + 3$, $p_2 = 2x^2 - 5x + 2$, and $p_3 = 2x^2 - 3x + 10$ are linearly independent in P_2.

[13] Suppose A is a 3×5 matrix, and the column space of A is 2-dimensional.

(a) What is the rank of A?
(b) What is the dimension of the row space of A?
(c) What is the dimension of the null space of A?
(d) What is the dimension of the null space of A^T?
(e) Is the row space of A a subspace of $\mathbb{R}^3$ or $\mathbb{R}^5$?

[14] Consider the vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ -3 \\ 4 \\ 0 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} -2 \\ 6 \\ -8 \\ 0 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 3 \\ 2 \\ -5 \\ 1 \end{bmatrix}, \vec{v}_4 = \begin{bmatrix} 5 \\ -4 \\ 3 \\ 1 \end{bmatrix}$.

Let $V = \text{span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$

(a) Find a basis for V.

(b) What is the dimension of V?

(c) Is $\vec{w} = \begin{bmatrix} 1 \\ -25 \\ 38 \\ -2 \end{bmatrix}$ a vector in $V = \text{span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$?

[15] Let $A = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 0 & 3 & 1 \\ 1 & 1 & 5 & 2 \end{bmatrix}$. Find a basis for the column space of A.

[16] Find the characteristic polynomial, eigenvalues, and a basis for each eigenspace for $A = \begin{bmatrix} 4 & 2 \\ 2 & 7 \end{bmatrix}$.

[17] Find the determinant of $\begin{bmatrix} 8 & 3 & 0 & 5 \\ 5 & 3 & 1 & -2 \\ 2 & 2 & 0 & 5 \\ 12 & 6 & 2 & -4 \end{bmatrix}$.

[18] Consider the vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} -5 \\ -2 \\ 1 \end{bmatrix}$.

(a) Verify that the vectors form an orthogonal set.

(b) Write the vector $\vec{w} = \begin{bmatrix} -14 \\ -6 \\ 8 \end{bmatrix}$ as a linear combination of $\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$.

[19] Diagonalize the matrix $A = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 3 & 2 \\ 0 & 1 & 4 \end{bmatrix}$ (if possible).