SI Session: Exam II Review
Mondays: 3:00 PM – 4:30 PM
Tuesdays: 1:30 PM – 3:00 PM
Thursdays: 1:30 PM – 3:00 PM
Room 1239 SNAD

Prof. McCurdy : Linear Algebra
Fall 2008
SI Leader : Neil Jody

[1] Let $A = \begin{bmatrix} 0 & 2 & 5 \\ 1 & 1 & 2 \\ 2 & 3 & 6 \end{bmatrix}$.

(a) Calculate $2A + A^T$.

(b) Calculate A^{-1}.

[2] Let $T:\mathbb{R}^3 \to \mathbb{R}^2$ be defined by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x-z \\ 2y \end{bmatrix}$. Show that T is a linear transformation.

[3] Let $A = \begin{bmatrix} 2 & x \\ 3 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 5 \\ 4 & 1 \end{bmatrix}$. For what value of x will $AB = \begin{bmatrix} 38 & 21 \\ 11 & 20 \end{bmatrix}$?

[4] Let $S = \{(a, b, 2a+b) | a, b \in \mathbb{R}\}$. Prove that S is a subspace of $\mathbb{R}^3$.

[5] Let S be a set of all ordered pairs $\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ with vector addition defined by

$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} u_2 + v_2 \\ u_1 + v_1 \end{bmatrix}$ and scalar multiplication defined by $c\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} cu_1 \\ u_2 \end{bmatrix}$.

Determine whether S satisfies Axiom 8 : $(c+d)\vec{u} = c\vec{u} + d\vec{u}$

[6] Decide whether the polynomials $p_1 = x^2 - 2x + 3$, $p_2 = 2x^2 - 5x + 2$, and $p_3 = 2x^2 - 3x + 10$ are linearly independent in P_2.

[7] Suppose A is a 3×5 matrix, and the column space of A is 2-dimensional.

(a) What is the rank of A?
(b) What is the dimension of the row space of A?
(c) What is the dimension of the null space of A?
(d) What is the dimension of the null space of A^T?
(e) Is the row space of A a subspace of $\mathbb{R}^3$ or $\mathbb{R}^5$?

[8] Consider the vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ -3 \\ 4 \\ 0 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} -2 \\ 6 \\ -8 \\ 0 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 3 \\ 2 \\ -5 \\ 1 \end{bmatrix}, \vec{v}_4 = \begin{bmatrix} 5 \\ -4 \\ 3 \\ 1 \end{bmatrix}$.

Let $V = \text{span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$

(a) Find a basis for V.

(b) What is the dimension of V?

(c) Is $\vec{w} = \begin{bmatrix} 1 \\ -25 \\ 38 \\ -2 \end{bmatrix}$ a vector in $V = \text{span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$?

[9] Let $A=\begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 0 & 3 & 1 \\ 1 & 1 & 5 & 2 \end{bmatrix}$. Find a basis for the column space of A.

[10] Let $T:\mathbb{R}^3 \to \mathbb{R}^2$ be defined by $T(\vec{x})=\begin{bmatrix} -2 & 5 & 6 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$.

Find a basis for the kernel of T.

[11] Give an example of a set of vectors that span $\mathbb{R}^3$ but which are not linearly independent in $\mathbb{R}^3$.