SI Session: April 27th,28th & 29th, 2009 Mondays: 4:50 PM – 6:20 PM Tuesdays: 1:30 PM – 3:00 PM Wednesdays: 4:50 PM – 6:20 PM Room 1245 SNAD

Prof. Stockton : Calculus III Spring 2009 SI Leader : Neil Jody

[1] Determine if the following vector field is conservative. If it is, find a potential function for the vector field.

(a)
$$\vec{F}(x,y) = \langle 2x - y^2, y^3 - 2xy \rangle$$

(b) $\vec{F}(x,y,z) = e^z \left(y\hat{i} + x\hat{j} + \hat{k} \right)$
(c) $\vec{F}(x,y,z) = y^2 z^3 \hat{i} + 2xy z^3 \hat{j} + 3x y^2 z^2 \hat{k}$
(d) $\vec{F}(x,y,z) = \frac{x}{x^2 + y^2} \hat{i} + \frac{y}{x^2 + y^2} \hat{j} + \hat{k}$

[2] Find curl \vec{F} for the vector field at the given point.

(a)
$$\vec{F}(x, y, z) = x^2 z \hat{i} - 2x z \hat{j} + y z \hat{k}$$
, $(2, -1, 3)$
(b) $\vec{F}(x, y, z) = e^{-xyz} (\hat{i} + \hat{j} + \hat{k})$, $(3, 2, 0)$

[3] Find the divergence for the vector field \vec{F} .

(a)
$$\vec{F}(x, y, z) = xe^{x}\hat{i} + ye^{y}\hat{j}$$

(b)
$$\vec{F}(x, y, z) = \ln(x^2 + y^2)\hat{i} + xy\hat{j} + \ln(y^2 + z^2)\hat{k}$$

- [4] Let *C* be the curve in space parametrized by $\vec{r}(t) = (t, t^2, 1-t)$ for $t \in [0, 2]$. An object moving along *C* with the orientation prescribed by \vec{r} is acted on by the force field $\vec{F}(x, y, z) = \langle x, x - z, 2y \rangle$. Calculate the work done by \vec{F} .
- [5] A smooth curve *C* lying in the *xy*-plane begins at the point (1, 1) and ends at (3, 2) (see figure). Calculate the following line integral:

$$\int_{C} (2x - y^2) dx + (y^3 - 2xy) dy$$

[6] Compute the following line integral: $\int_{C} (x^2 + y^2) dx - xy dy$ where *C* is the

upper arc of the circle $x^2 + y^2 = 4$, traversed counterclockwise.

[7] Evaluate
$$\int_{C} yds$$
 where *C* is the path shown below: (1,2)

[8] Use Green's Theorem to calculate the following line integral: $\int_{C} (x^2 + xy)dx + x^2y^3dy \text{ where } C \text{ is the boundary of the region } D \text{ shown}$

below traversed counterclockwise.



[9] Use Green's Theorem to evaluate the line integral $\int_C y^2 dx + 6xy dy$ where C

is the path from (0,0) to (1,0) along y = 0, from (1,0) to (1,1) along x = 1, and from (1,1) to (0,0) along $y = \sqrt{x}$

- [10] Let *S* represent the portion of the plane x + y + 2z = 4 lying inside the cylinder $x^2 + z^2 = 1$.
 - (a) Give a parametrization of *S*.
 - (b) Calculate the surface area of *S*.

(c) Evaluate
$$\iint_{S} (4-y) dS$$