MATH 2415 Calculus III SI Session 10/19/2011

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Wednesdays Rm 1245 03:00 PM - 05:00 PM and 05:00 PM - 07:00 PM

1. Find all points on the surface $xy - 2x + z^2 = 8$ at which the plane tangent to the surface is parallel to the plane with equation $x + y - 2z = 3$

2. Find the absolute extrema of the function over the region R. In each case, R contains the boundaries.
   (a) $f(x, y) = (2x - y)^2$, R : The triangular region in the xy-plane with vertices (2,0), (0,1) and (1,2).
   (b) $f(x, y) = x^2 - 4xy + 5$, $R = \{(x, y) : 0 \leq x \leq 4, 0 \leq y \leq \sqrt{x}\}$
   (c) $f(x, y) = \frac{4xy}{(x^2+1)(y^2+1)}$, $R = \left\{(x, y) : x \geq 0, y \geq 0, x^2 + y^2 \leq 1\right\}$

3. For each of the following functions, find all local extrema and saddle points.
   (a) $f(x, y) = x^3 - 4xy + 2y^2 - 1$ (b) $g(x, y) = x^2 - 4xy + y^3 + 4y$

4. Find the maximum and minimum values of the function $g(x, y) = x^2 + y^2 - x - y + 1$ on the unit disk $x^2 + y^2 \leq 1$

5. The function $f(x, y, z) = 2x^2 + y^2 + 3z^2$ has a minimum value on the plane $2x - 3y + 4z = 49$. Find this minimum value.

6. A spider living in a two-dimensional world finds itself in a toxic environment. The toxicity at (x, y) is given by the function $T(x, y) = 4x^2 - 4xy + y^2$
   (a) If the spider is at the point (2, 1) , in which direction should it move in order to *lower* the toxicity the fastest?
   (b) Use Lagrange multipliers to determine the points along the parabola $y = x^2$ at which the toxicity is the lowest.

7. Use Lagrange multipliers to find the *maximum* value of the function $f(x, y, z) = 2xy + 3z^2$ on the sphere $x^2 + y^2 + z^2 = 4$.

8. Find the absolute maximum and minimum values of the function $f(x, y) = x^2y - x^2 - y + 1$ on the triangular region with vertices (0,0), (2,4) and (2,0).