Wednesdays Rm 1245
03:00 PM - 05:00 PM and 05:00 PM - 07:00 PM

1. Find both first partial derivatives.
(a) $f(x, y)=x^{2}-3 y^{2}+7$
(b) $z=2 y^{2} \sqrt{x}$
(c) $f(x, y)=\ln \left(x^{2}+y^{2}\right)$
(d) $z=\sin (3 x) \cos (3 y)$
(e) $z=\cos \left(x^{2}+y^{2}\right)$
2. Evaluate $f_{x}$ and $f_{y}$ at the given point.
(a) $f(x, y)=\arccos (x y),(0, \sqrt{5})$
(b) $f(x, y)=\frac{6 x y}{\sqrt{4 x^{2}+5 y^{2}}},(1,1)$
3. For $f(x, y)$, find all values of $x$ and $y$ such that $f_{x}(x, y)=0$ and $f_{y}(x, y)=0$ simultaneously.
(a) $f(x, y)=3 x^{3}-12 x y+y^{3}$
(b) $f(x, y)=\ln \left(x^{2}+y^{2}+1\right)$
4. Find $w_{s}$ and $w_{t}$ using the appropriate chain rule, and evaluate each partial derivative at the given values of $s$ and $t$.
(a) $w=y^{3}-3 x^{2} y ; x=\mathrm{e}^{s}, y=\mathrm{e}^{t}$, at the point $s=0$ and $t=1$
(b) $w=\sin (2 x+3 y) ; x=s+t, y=s-t$, at the point $s=0$ and $t=\frac{\pi}{2}$
5. If $w=x y^{2}$ and $x=2 s-t, y=t^{2}$, using the Chain Rule to find $w_{t}$ at the point $(-1,2)$ in the $s t$-plane.
6. Suppose $w$ is a function of $r$ and $s$ where $r=x y+y z^{2}, s=\sin (y)+\mathrm{e}^{x z}$. Use the information given below to compute $w_{y}(-1,0,0)$ : $w_{r}(0,1)=2, w_{s}(0,1)=5$
7. If $p$ is a differentiable function of $u, v$ and $w$, and if $u=x-y, v=y-z$, and $w=z-x$, show that $p_{x}+p_{y}+p_{z}=0$.
8. Find the directional derivative of the function at $P$ in the direction of $\mathbf{v}$.
(a) $g(x, y)=\arccos (x y), P(1,0), \mathbf{v}=\hat{i}+5 \hat{j}$
(b) $h(x, y)=\mathrm{e}^{-\left(x^{2}+y^{2}\right)}, P(0,0), \mathbf{v}=\hat{i}+\hat{j}$
(c) $h(x, y, z)=x y z, P(2,1,1), \mathbf{v}=2 \hat{i}+\hat{j}+2 \hat{k}$
9. Find the directional derivative of the function at $P$ in the direction of $Q$.
(a) $f(x, y)=\cos (x+y), P(0, \pi), Q\left(\frac{\pi}{2}, 0\right)$
(b) $g(x, y, z)=x y \mathrm{e}^{z}, P(2,4,0), Q(0,0,0)$
10. Find the direction of maximum increase of the function the given point.
(a) $g(x, y)=y \mathrm{e}^{-x^{2}},(0,5)$
(b) $w=x y^{2} z^{2},(2,1,1)$
11. The surface of a mountain is modeled by the equation $h(x, y)=5000-0.001 x^{2}-0.004 y^{2}$. A mountain climber is at the point $(500,300,4390)$. In what direction should the climber move in order to ascend at the greatest rate?
12. A ground-dwelling cold hairy spider wants to get warm. The temperature at the point $(x, y)$ is given by $T(x, y)=x^{2}-x y+2 y^{2}$ (degrees Fahrenheit). If the spider is at the point $(2,3)$, in which direction should it move to increase its temperature the fastest?
13. At time $t=0$ (seconds), a particle is ejected from the surface $x^{2}+y^{2}-z^{2}=-1$ at the point $(1,1, \sqrt{3})$ in a direction normal to the surface at a speed of 10 units per second. When and where does the particle cross the $x y$-plane?
14. Find the directional derivative of the function $f(x, y, z)=3 x z-2 x y^{2}$ at the point $(-1,1,2)$ in the direction from $(-1,1,2)$ to $(1,3,3)$.
15. Find the gradient and the direction of maximum decrease at the given point.
(a) $z=\mathrm{e}^{-x} \cos (y),\left(0, \frac{\pi}{4}\right)$
(b) $z=\frac{x^{2}}{x-y},(2,1)$
16. Find an equation of the tangent plane at the given point.
(a) $z=x^{2}-2 x y+y^{2},(1,2,1)$
(b) $x=y(2 z-3) 4,(4,4,2)$
17. Find an equation of the tangent plane and parametric equations of the line normal to the surface at the given point.
(a) $z=-9+4 x-6 y-x^{2}-y^{2},(2,-3,4)$
(b) $z=\sqrt{9-x^{2}-y^{2}},(1,2,2)$
