MATH 2415	Calculus III SI Session	09/07/11
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Wednesdays Rm 1245		03:00 PM - 05:00 PM and 05:00 PM - 07:00 PM

1. Find the vectors **u** and **v** whose initial and terminal points are given. Show that **u** and **v** are equivalent.
   (a)$\mathbf{u} : (-4, 0), (1, 8)$, $\mathbf{v} : (2, -1), (7, 7)$
   (b)$\mathbf{u} : (-4, -1), (11, -4)$, $\mathbf{v} : (10, 13), (25, 10)$

2. Find the vector **v** where $\mathbf{u} = (2, -1)$ and $\mathbf{w} = (1, 2)$. Illustrate the vector operations geometrically.
   (a) $\mathbf{v} = \mathbf{u} + \mathbf{w}$ (b) $\mathbf{v} = 5\mathbf{u} - 3\mathbf{w}$

3. Find the following: $\|\mathbf{u}\|$, $\|\mathbf{v}\|$, $\|\mathbf{u} + \mathbf{v}\|$, $\left\|\frac{\mathbf{u}}{\|\mathbf{u}\|}\right\|$, $\left\|\frac{\mathbf{u}+\mathbf{v}}{\|\mathbf{u}+\mathbf{v}\|}\right\|$.
   (a) $\mathbf{u} = (0, 1)$, $\mathbf{v} = (3, -3)$

4. Find a unit vector(1)parallel to and (2)normal to the graph of $f(x)$ at the given point.
   (a) $f(x) = \sqrt{25 - x^2}$, $(3, 4)$
   (b)$f(x) = \tan x$, $(\frac{\pi}{4}, 1)$

5. Let $\mathbf{u}_0 = (x_0, y_0)$, $\mathbf{u} = (x, y)$, and $c > 0$. Describe the set of all points $P(x, y)$ such that $\|\mathbf{u} - \mathbf{u}_0\| = c$.

6. Find the coordinates of the point.
   (a) The point is located seven units in front of the $yz - plane$, two units to the left of the $xz - plane$, and one unit below the $xy - plane$.
   (b) The point is located in the $yz - plane$, three units to the right of the $xz - plane$, and two units above the $xy - plane$.

7. What is the $x-$coordinate of any point in the $yz - plane$?

8. Find the distance between the given points.
   (a) $(-2, 3, 2)$, $(2, -5, -2)$, (b) $(2, 2, 3)$, $(4, -5, 6)$

9. Write the equation of the sphere in standard form. State the center and the radius.
   (a) $x^2 + y^2 + z^2 + 9x - 2y + 10z + 19 = 0$

10. Find the vector **v** with magnitude $\sqrt{5}$ and direction of $\mathbf{u} = (-4, 6, 2)$.

11. Prove the following version of the triangle inequality: $|\,\|\mathbf{a}\| - \|\mathbf{b}\|\,| \leq \|\mathbf{a} - \mathbf{b}\|$. *Hint*: $\mathbf{a} = (\mathbf{a} - \mathbf{b}) + \mathbf{b}$.

12. Let **a** and **b** be nonzero vectors such that $\|\mathbf{a} - \mathbf{b}\| = \|\mathbf{a} + \mathbf{b}\|$.
    (a) What can you conclude about the parallelogram generated by **a** and **b**?
    (b) Show that if $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, then $a_1b_1 + a_2b_2 + a_3b_3 = 0$.

13. (a) Show that if **a** and **b** have the same direction, then $\|\mathbf{a} + \mathbf{b}\| = \|\mathbf{a}\| + \|\mathbf{b}\|$.
    (b) Does this equation necessarily hold if **a** and **b** are only parallel?

14. Find α given that $\|\alpha\mathbf{i} + (\alpha - 1)\mathbf{j} + (\alpha + 1)\mathbf{k}\| = 2$.

15. Find the angle θ between the vectors.
    (a) $\mathbf{u} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$, $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j}$, (b) $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, $\mathbf{v} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$

16. Determine whether **u** and **v** are orthogonal, parallel, or neither.
    (a) $\mathbf{u} = -2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$, $\mathbf{v} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$, (b) $\mathbf{u} = (\cos\theta, \sin\theta, 1)$, $\mathbf{v} = (\sin\theta, -\cos\theta, 0)$

17. Find the projection of **u** onto **v** and the vector component of **u** orthogonal to **v**.
    (a) $\mathbf{u} = (2, -3)$, $\mathbf{v} = (3, 2)$, (b) $\mathbf{u} = (1, 0, 4)$, $\mathbf{v} = (3, 0, 2)$