MATH 2415 Calculus III SI Session 10/31 and 11/02/2011

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Wednesdays Rm 1245 03:00 PM - 05:00 PM and 05:00 PM - 07:00 PM

1. Let $f(x,y) = 4x^2 + 9y^2$.
   (a) Sketch the level curve of f containing the point (0,-2).
   (b) Find the unit vector which points in the direction in which f decreases most rapidly at (0,-2).

2. Find an equation of the plane tangent to the surface $x^2 + yz^3 = 4$ at the point (-1,3,1).

3. Find the directional derivative of the function $f(x,y,z) = xyz$ at the point (1,2,-2) in the direction from (1,2,-2) to (-1,0,-1).

4. Find the absolute extrema of the function $f(x,y) = x^2 + y^2 - 6y$ on the closed region bounded by the graphs $y = 4 - x^2$ and $x + y = 2$.

5. Use Lagrange Multipliers to find the maximum and minimum of the function $f(x,y,z) = x + y + z$ on the sphere $x^2 + y^2 + z^2 = 4$.

6. Let $f(x,y,z) = x\ln(y) + y^2\sin(xz)$. Calculate f_{yxy}.

7. Evaluate the integral $\int_1^2 \int_0^{\sqrt{4-x^2}} \frac{x}{x^2+y^2} dydx$ by first converting to polar coordinates.

8. Let $z = x^2 + xy$, $x = rs + 2t$, $y = r^2 - st$. Calculate z_r when $r = 1$, $s = -2$ and $t = 3$.

9. Let $f(x,y) = 6x^2 - 2x^3 + 3y^2 + 6xy$. Find all relative extrema and saddle points for f.

10. Let D be the triangular region in the $xy-$plane bounded by the graphs of $x + y = 6$, $y = 2x$ and $5y = x$. Using the change of variables $x = 5u + v$ and $y = u + 2v$, express $\int\int_D (5y - x)\,\mathrm{e}^{y-2x}\,dxdy$ ans an iterated integral in the variables u and v. Do not evaluate the integral.

11. Let D be the region in the $xy-$plane bounded by the $x-$axis, the $y-$axis, the line $y = 1$ and the curve $y = \ln(x)$. Express $\int\int_D f(x,y)dA$ as an iterated integral. Do not evaluate the integral.

12. Express $\int_0^1 \int_3^{4-x^2} f(x,y)dydx$ as an iterated integral with the reverse order of integration.

13. Let D be the region in the $xy-$plane bounded by the lines $y = 3x$, $2y = x$ and $x = 4$. Using the change of variables $x = u - 2v$ and $y = 3u - v$, evaluate $\int\int_D (y - 3x)dydx$.

14. Let D be the region in the $xy-$plane bounded by the lines $x + 2y = 2$, $y = x + 1$ and $y = -2x + 4$. Use the change of variables $x = u + 2v$ and $y = u - v + 1$ to evaluate the integral $\int\int_D (x - y)dA$.

15. Evaluate the integral $\int_{-2}^2 \int_0^{\sqrt{4-y^2}} \sin(x^2 + y^2)dxdy$ by first converting to polar coordinates.

16. Let D be the region in the $xy-$plane bounded on the left by the $y-$axis, above by the graph of $x^2 + y^2 = 4$ and below by the line $y = 1$. Evaluate $\int\int_D \frac{1}{(x^2+y^2)^{3/2}} dxdy$ by converting to polar coordinates.