SI Session: Final Exam Review
Saturday, December 6th
12:00 PM – 2:00 PM
Monday, December 8th
1:30 PM – 3:00 PM
& 4:50 PM – 6:20 PM
Room 1239 SNAD

Prof. Stockton : Calculus III
Fall 2008
SI Leader : Neil Jody

[1] Let $\vec{u} = \langle 1,3,-4 \rangle$ and $\vec{v} = \langle 2,-2,1 \rangle$.

(a) Calculate $proj_{\vec{u}}\vec{v}$.

(b) Calculate $\vec{u} \times \vec{v}$.

(c) Calculate $\left(2\vec{u} - 3\vec{v}\right) \cdot \left(\vec{u} + 4\vec{v}\right)$.

(d) Calculate the area of the parallelogram with adjacent sides $\vec{u}$ and $\vec{v}$.

[1] (continued from previous page)

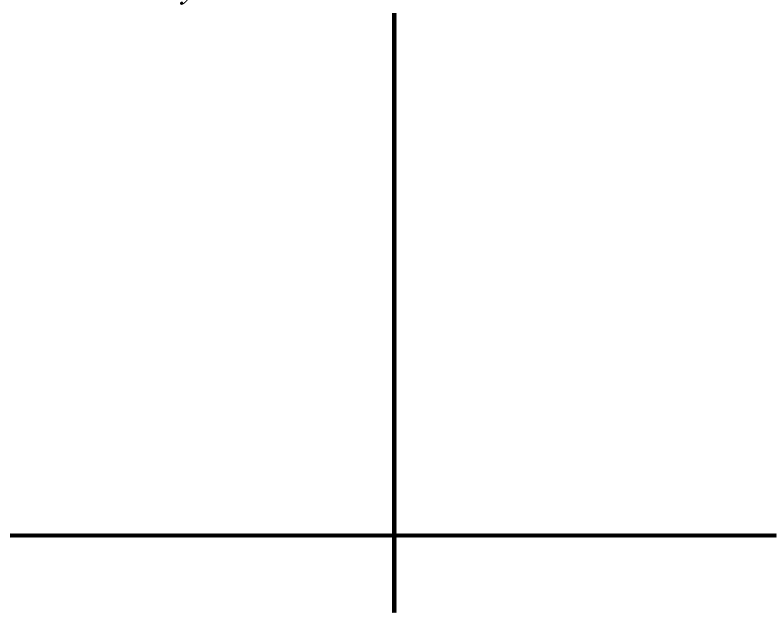
(e) Find a vector that has the direction of $\vec{u}$ and the length of $\vec{v}$.

(f) Determine the value of c so that the vector $\langle c,1,-2\rangle$ lies in the plane of $\vec{u}$ and $\vec{v}$.

[2] Find an equation of the plane tangent to the surface $x^2 + yz^3 = 4$ at the point $(-1,3,1)$.

[3] Find the directional derivative of the function $f(x,y,z) = xyz$ at the point $(1,2,-2)$ in the direction from $(1,2,-2)$ to $(-1,0,-1)$.

[4] Find the absolute extrema of the function $f(x,y) = x^2 + y^2 - 6y$ on the closed region bounded by the graphs $y = 4 - x^2$ and $x + y = 2$.



[5] Use Lagrange Multipliers to find the maximum and minimum of the function $f(x,y,z) = x + y + z$ on the sphere $x^2 + y^2 + z^2 = 4$.

[6] Let $f(x,y) = 6x^2 - 2x^3 + 3y^2 + 6xy$. Find all relative extrema and saddle points for f.

[7] Let D be the region in the xy-plane bounded on the left by the y-axis, above by the graph of $x^2 + y^2 = 4$ and below by the line $y = 1$.

Evaluate $\iint_D \frac{1}{\left(x^2+y^2\right)^{\frac{3}{2}}}\,dxdy$ by converting to polar coordinates.

[8] Find parametric equations of the line tangent to the curve generated by $\vec{r}(t) = \left(e^{t-2}, t^2 - 1, \sqrt{t+2}\right)$ at the point $(1,3,2)$.

[9] The position of a particle at time t is given by the function $\vec{r}(t) = \left(\sin t, t^2 + 4t, -\cos t\right)$.

(a) At what time(s), if any, is the speed of the particle equal to 2 ?

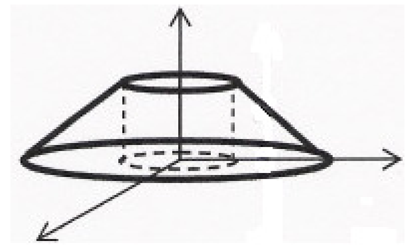
(b) At what time(s), if any, will the velocity and acceleration vectors be orthogonal?

[10] Find the length of the curve generated by $\vec{r}(t) = \left(2t, t^2, \frac{1}{3}t^3\right)$ where $0 \le t \le 3$.

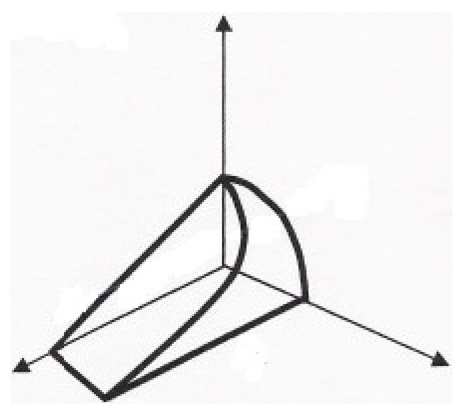
[11] Let C the line segment generated by $\vec{r}(t) = (2-t, 2t)$ for $0 \le t \le 1$. Evaluate the following line integral: $\int_C y\,dx - 2x\,dy$

[12] Use Green's Theorem to evaluate the line integral $\int_C -y^3 dx + x^3 dy$ where C is the circle of radius 2 centered at the origin oriented counterclockwise.

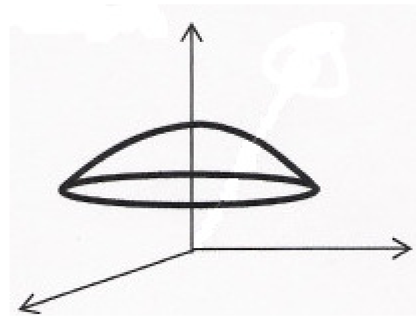
[13] Let Q be the space lying below the inverted cone $z = 2 - \sqrt{x^2 + y^2}$, above the xy-plane, and outside the cylinder $x^2 + y^2 = 1$ (see figure).
Express $\iiint_Q \left(x^2 + y^2 + z^2\right) dxdydz$ as an iterated integral in cylindrical coordinates. Do not evaluate the integral.



[14] Express as a triple iterated integral the volume of the solid Q in the first octant bounded by the coordinate planes and the graphs of $y^2 + z^2 = 4$ and $2z + x = 4$ (see diagram below):



[15] Let Q be the region inside the sphere $x^2 + y^2 + z^2 = 4$ and above the plane $z = 1$ (see figure). Express the volume of Q as triple iterated integral in spherical coordinates. Do not evaluate the integral.



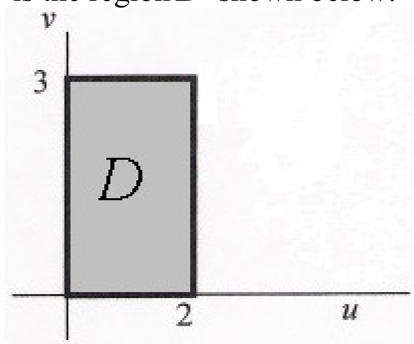
[16] Let C be a smooth curve in the xy-plane from $(1,0)$ to $(3,-1)$.
Evaluate the following line integral: $\int_C (2x + y - 3)\,dx + x\,dy$

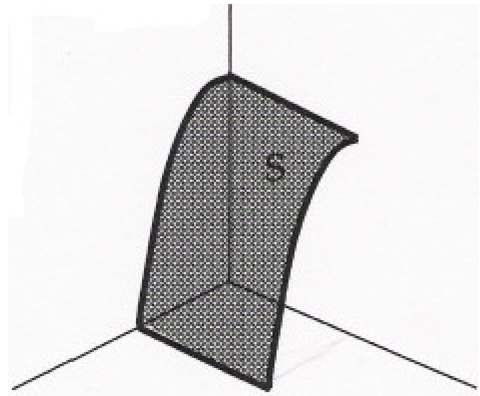
[17] Let S be the portion of the cylinder $z = 4 - x^2$ lying in the first octant to the left of the plane $y = 3$ (see diagram).

A parametrization of S is

$\vec{r}(u,v) = (u, v, 4 - u^2)$

where the domain of $\vec{r}$ is the region D shown below:





(a) Express the surface area of S as an iterated integral in the variables u and v.
Do not evaluate the integral.

(b)Express the surface integral $\iint_S xyz \, dS$ as an iterated integral in the variables u and v.

Do not evaluate the integral.

[18] Rewrite the integral $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{4-x^2-y^2} f(x,y,z) \, dzdydx$ in the order $dxdzdy$.

[19] Let $\vec{F}(x,y,z)=\langle x^2yz, 2xy, xz^3\rangle$. Calculate

(a) div $\vec{F}$

(b) curl $\vec{F}$