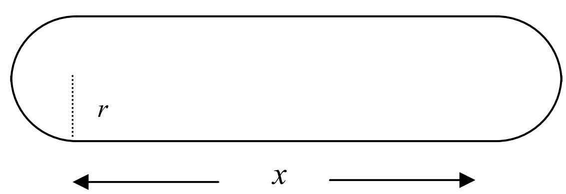
SI Session: July 7th
Monday
12:30 PM – 2:30 PM
Room 1229

Prof. Stockton : Calculus I
Summer I 2008
SI Leader : Neil Jody

[1] Express $\lim_{\|P\|\to 0} \sum_{i=1}^{n} \left(2c_i \sqrt{(c_i)^2 + 4} \right) \Delta x_i$ as a definite integral over the interval [1,5] where c_i is any point in the ith subinterval.

[2] Calculate $\frac{dy}{dx}$ if $y = \int_{x^2}^{x^3} \sqrt{1+t^3}\, dt$.

[3] You are designing an athletic field in the shape of a rectangle x meters long capped at two ends by semicircular regions of radius r. The boundary of the field is to be a 400 meter track. What values of x and r will give the *rectangle* its greatest area?



[4] Find the exact area of the region below the graph of $y = 4 - x^2$, above the x-axis and between the lines $x = -2$ and $x = 1$, by taking the limit of a Riemann sum.

[5] Find the function f with the following properties:

(i) $f''(x) = 6x$ and

(ii) its graph contains the point (2, 9) and has a horizontal tangent there.

[6] Let $f(x) = x^3 + 2x^2 - 1$. Suppose $x = 2$ and $dx = 0.02$.

(a) Calculate Δy

(b) Calculate dy.

[7] Evaluate each integral.

(a) $\int \frac{2x^2 - x + 3}{\sqrt{x}}\, dx$

(b) $\int \frac{3x}{4 + x^2}\, dx$

(c) $\int \frac{3x + 6}{\sqrt[3]{x^2 + 4x - 3}}\, dx$

(d) $\int \sec^2\left(\frac{x}{3}\right)\tan^2\left(\frac{x}{3}\right)\, dx$

(e) $\int \frac{3\cos\left(4+\frac{1}{x}\right)}{x^2}\, dx$

(f) $\int_{0}^{\pi/3} \sin x \cos^2 x \, dx$

(g) $\int_{-2}^{3} |2x - 4| \, dx$

[8] Find $\frac{dy}{dx}$ in each case.

(a) $y = \ln\left(\frac{x^2 \sin x\sqrt{2x+3}}{(x^2+4)\ln x}\right)$

(b) $\ln\left(xy^2\right) = \cos x$

(c) $y = \tan(\ln \sqrt{x})$

(d) $y = \ln(\sin(3x))$

1. $\frac{d}{dx}\left[c\, f(x)\right] = c\frac{d}{dx}\left[f(x)\right]$

2. $\frac{d}{dx}\left[f(x) \pm g(x)\right] = \frac{d}{dx}\left[f(x)\right] \pm \frac{d}{dx}\left[g(x)\right]$

3. $\frac{d}{dx}\left[\mathrm{f}(x)g(x)\right] = \mathrm{f}'(x)g(x) + \mathrm{f}(x)g'(x)$

4. $\frac{d}{dx}\left[\frac{\mathrm{f}(x)}{g(x)}\right] = \frac{\mathrm{f}'(\mathrm{x})g(x) - \mathrm{f}(\mathrm{x})g'(x)}{\left(g(x)\right)^2}$

5. $\int cf(x)\,dx = c\int f(x)\,dx$

6. $\int \left[f(x) \pm g(x)\right]dx = \int f(x)\,dx \pm \int g(x)\,dx$

7. $\frac{d}{dx}\left[\log_b x\right] = \frac{1}{x}\cdot\frac{1}{\ln b}$

8. $\int x^n dx = \frac{x^{n+1}}{n+1} + C, \text{if } n \neq -1$

9. $\int x^n dx = \ln|x| + C, \text{if } n = -1$

10. $\int \sin x\,dx = -\cos x + C$

11. $\int \cos x\,dx = \sin x + C$

12. $\int \sec^2 x\,dx = \tan x + C$

13. $\int \csc^2 x\,dx = -\cot x + C$

14. $\int \sec x \tan x\,dx = \sec x + C$

15. $\int \csc x \cot x\,dx = -\csc x + C$

16. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$

17. $\int \tan x\,dx = -\ln|\cos x| + C$

18. $\int \cot x\,dx = \ln|\sin x| + C$

19. $\int \csc x\,dx = \ln|\csc x - \cot x| + C$

20. $\int b^x dx = \frac{b^x}{\ln b} + C, (0 < b, b \neq 1)$

21. $\sin^2(\theta) + \cos^2(\theta) = 1$

22. $\tan^2(\theta) + 1 = \sec^2(\theta)$

23. $1 + \cot^2(\theta) = \csc^2(\theta)$

24. $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$

25. $\cos(2\theta) = \begin{cases} \cos^2(\theta) - \sin^2(\theta) \\ 2\cos^2(\theta) - 1 \\ 1 - 2\sin^2(\theta) \end{cases}$

For all real numbers y, and all positive numbers a and x, where $a \neq 1$:

$\log_b x = y \Leftrightarrow b^y = x$

For $x > 0, y > 0, a > 0, a \neq 1$, and any real number r :

26.
$\log_b x^r = r \cdot \log_b x$

$\log_b xy = \log_b x + \log_b y$

$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

For any positive real numbers x, a, and b, where $a \neq 1$ and $b \neq 1$:

$\log_b x = \frac{\log x}{\log b} = \frac{\ln x}{\ln b} = \frac{\log_a x}{\log_a b}$

1.) $\Delta x = \frac{b-a}{n}$

2.) the right endpoint of the k^{th} interval is $a + k\Delta x$.

3.) $S_n = \sum_{k=1}^{n} f(a + k\Delta x)\Delta x$

4.) Area $= \lim_{n\to\infty} S_n$

(a) $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

(b) $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

(c) $\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$

www.neiljody.com