SI Final Exam Review
Wednesday July 9th
12:30 PM – 3:30 PM
Room 1229

Prof. Stockton : Calculus I
Summer I 2008
SI Leader : Neil Jody

[1] Evaluate each limit, if it exists. Your answer should be a number, ∞, $-\infty$, or DNE.

(a) $\lim_{x \to 0} \dfrac{\sin 4x}{\sin 3x}$

(b) $\lim_{x \to 1^-} \dfrac{3x-3}{|x-1|}$

(c) $\lim_{x \to 1^-} g(x)$ where $g(x) = \begin{cases} \frac{x}{x+2} & \text{if } x \le 1 \\ \frac{-2}{x-1} & \text{if } x > 1 \end{cases}$

(d) $\lim_{x \to 1} \dfrac{\sqrt{x^2+3x}-2}{x-1}$

(e) $\lim_{x \to 3} \frac{\sin(x-3)}{x^2-9}$

(f) $\lim_{x \to 0} \frac{\frac{1}{x+2} - \frac{1}{2}}{x}$

(g) $\lim_{x \to \infty} \frac{\sin\sqrt{x}}{\sqrt{x}}$

(h) $\lim_{x \to \infty} \frac{3x^2-5x}{4-2x^3}$

(i) $\lim_{x\to-\infty} \frac{2x-3}{\sqrt{3+2x^2}}$

[2] Find the vertical and horizontal asymptotes of the function $f\left(x\right)=\frac{x^2+2x+1}{x^2+6x+5}$.

[3] A bionic man is programmed to run the 80-meter dash in such a way that the distance, s, run after t seconds is given by $s=\frac{1}{4}t^2+6t$ (meters). Find the runner's velocity when he crosses the finish line.

[4] Let $f(x) = \sqrt{x}$ .Find the largest real number $\delta > 0$ such that if $0 < |x - 4| < \delta$, then $|f(x) - 2| < 0.05$.

[5] Use the definition of derivative to find the derivative of $k(x) = \cos x$.

[6] Find the derivatives of each of the following functions. Do not simplify the result.

(a) $h(x) = \dfrac{\sqrt{2x+3}}{x^3 + 2x - 1}$

(b) $w(x) = \sin(\sec(\tan x))$

[7] Find $\frac{dy}{dx}$ in each case.

(a) $y = \log_x 5$

(b) $y = (\cos x)^x$

(c) $y = x^2 e^{3x}$

[8] Evaluate each integral.

(a) $\int \cos x e^{\sin x} dx$

(b) $\int_{0}^{\ln 3} \frac{e^{3x} - e^{x}}{e^{2x}} dx$

(c) $\int \frac{e^{-x}}{1+e^{-x}} dx$

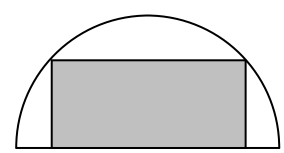
(d) $\int_{1}^{e} \frac{(\ln x)^4}{x} dx$

(e) $\int \frac{\sin(\ln x)}{x} dx$

(f) $\int_{0}^{\pi/6} \frac{\cos x}{2\sin x + 1} dx$

[8] For the curve given by $y^2 + x^2 y^3 + 11 = 4x$, find an equation of the tangent line at the point $(2,-1)$. Write the equation in the form $y = mx + b$.

[9] Find the dimensions of the rectangle of maximum area that can be inscribed in a semicircle of radius 4.



[10] A snowball is melting in the hot Pasadena sun. At the moment when the radius of the snowball is 3 inches, the volume of the snowball is decreasing at $4\pi\ \text{in}^3/\text{sec}$. How fast is the radius changing at that time?

[11] Let $f(x) = \sqrt[3]{x}(x-8)$. Find each of the following:

(a) open interval(s) on which f is increasing ____________________

(b) open interval(s) on which f is decreasing ____________________

(c) relative minima ____________________

(d) relative maxima ____________________

[12] Let $f(x) = x^4 - 6x^3$. Find each of the following:

(a) open interval(s) on which f is concave up ____________________

(b) open interval(s) on which f is concave down ____________________

(c) inflection points of f ____________________

[13] Find the absolute extrema of the function $f(x) = \frac{2x}{x^2+1}$ on the interval $[0,2]$.

1. $\frac{d}{dx}\left[c\, f(x)\right] = c\frac{d}{dx}\left[f(x)\right]$

2. $\frac{d}{dx}\left[f(x) \pm g(x)\right] = \frac{d}{dx}\left[f(x)\right] \pm \frac{d}{dx}\left[g(x)\right]$

3. $\frac{d}{dx}\left[\mathrm{f}(x)g(x)\right] = \mathrm{f}'(x)g(x) + \mathrm{f}(x)g'(x)$

4. $\frac{d}{dx}\left[\frac{\mathrm{f}(x)}{g(x)}\right] = \frac{\mathrm{f}'(\mathrm{x})g(x) - \mathrm{f}(\mathrm{x})g'(x)}{\left(g(x)\right)^2}$

5. $\int cf(x)\,dx = c\int f(x)\,dx$

6. $\int\left[f(x) \pm g(x)\right]dx = \int f(x)\,dx \pm \int g(x)\,dx$

7. $\int x^n dx = \frac{x^{n+1}}{n+1} + C, \text{if } n \neq -1$

8. $\int x^n dx = \ln|x| + C, \text{if } n = -1$

9. $\int \sin x\,dx = -\cos x + C$

10. $\int \cos x\,dx = \sin x + C$

11. $\int \sec^2 x\,dx = \tan x + C$

12. $\int \csc^2 x\,dx = -\cot x + C$

13. $\int \sec x \tan x\,dx = \sec x + C$

14. $\int \csc x \cot x\,dx = -\csc x + C$

15. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$

16. $\int \tan x\,dx = -\ln|\cos x| + C$

17. $\int \cot x\,dx = \ln|\sin x| + C$

18. $\int \csc x\,dx = \ln|\csc x - \cot x| + C$

19. $\int b^x dx = \frac{b^x}{\ln b} + C, (0 < b, b \neq 1)$

20. $\sin^2(\theta) + \cos^2(\theta) = 1$

21. $\tan^2(\theta) + 1 = \sec^2(\theta)$

22. $1 + \cot^2(\theta) = \csc^2(\theta)$

23. $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$

24. $\cos(2\theta) = \begin{cases} \cos^2(\theta) - \sin^2(\theta) \\ 2\cos^2(\theta) - 1 \\ 1 - 2\sin^2(\theta) \end{cases}$

For all real numbers y, and all positive numbers a and x, where $a \neq 1$:

$$\log_b x = y \Leftrightarrow b^y = x$$

For $x > 0, y > 0, a > 0, a \neq 1$, and any real number r :

25.

$$\log_b x^r = r \cdot \log_b x$$

$$\log_b xy = \log_b x + \log_b y$$

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

For any positive real numbers x, a, and b, where $a \neq 1$ and $b \neq 1$:

$$\log_b x = \frac{\log x}{\log b} = \frac{\ln x}{\ln b} = \frac{\log_a x}{\log_a b}$$

1.) $\Delta x = \frac{b-a}{n}$

2.) the right endpoint of the k^{th} interval is $a + k\Delta x$.

3.) $S_n = \sum_{k=1}^{n} f(a + k\Delta x)\Delta x$

4.) $\text{Area} = \lim_{n\to\infty} S_n$

(a) $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

(b) $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

(c) $\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$

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