

Part One: To be done without the use of a calculator.

[1] Evaluate each limit, if it exists. Your answer should be a number,  $\infty$ ,  $-\infty$ , or DNE .

(a)  $\lim_{x \rightarrow -3} \frac{x^2 + 2x}{|3x - 1|}$

(b)  $\lim_{x \rightarrow -\infty} \frac{\sqrt{5x^2 + 1}}{2x - 3}$

(c)  $\lim_{x \rightarrow 2^-} \frac{|x - 2|}{2x - 4}$

(d)  $\lim_{x \rightarrow 5} \frac{x^2 - 4x - 5}{x^2 - 8x + 15}$

[1] (continued)

$$(e) \lim_{x \rightarrow -\infty} \frac{|x+1|}{3x-1}$$

$$(f) \lim_{x \rightarrow 2^+} f(x) \text{ where, } f(x) = \begin{cases} x+3 & \text{if } x \leq 2 \\ 2x-1 & \text{if } x > 2 \end{cases}$$

$$(g) \lim_{x \rightarrow 0} \frac{\sin 3x}{2x}$$

$$(h) \lim_{x \rightarrow +\infty} \frac{x^3 + x^2 + 5x + 4}{3 - x^2}$$

$$(i) \lim_{x \rightarrow 4^-} \frac{3-2x}{x-4}$$

$$(j) \lim_{x \rightarrow 2} \frac{1 - \sqrt{2x-3}}{x-2}$$

[1] (continued)

$$(k) \lim_{x \rightarrow 0} \frac{\sin x}{1 + \cos x}$$

$$(l) \lim_{h \rightarrow 0} \frac{\frac{1}{x+h-3} - \frac{1}{x-3}}{h}$$

[2] Let  $f(x) = \frac{\sin x}{x^2 + 2x}$ . Find the discontinuities of  $f$  and determine if each is removable or nonremovable.

[3] Find the vertical and horizontal asymptotes of the function  $f(x) = \frac{x^2 + 2x + 1}{x^2 + 6x + 5}$ .

Part Two: Calculators permitted.

[4] Use the *definition of the derivative* to find the derivative of  $f(x) = \sqrt{3-x}$ .

[5] Let  $f(x) = \begin{cases} x^4 - 2x & \text{if } x < 1 \\ 2x - 3 & \text{if } x \geq 1 \end{cases}$

(a) Determine if  $f$  is continuous at  $x = 1$ .

(b) Determine if  $f$  is differentiable at  $x = 1$ .

[6] Let  $f(x) = 4x - 1$ . Find the Largest real number  $\delta > 0$  such that if  $0 < |x - 1| < \delta$ , then  $|f(x) - 3| < 0.02$ .

[7] Find the derivatives of each of the following functions. Do not simplify the result.

(a)  $f(x) = \frac{x^3 - 2x + \sqrt{x}}{x^2}$

(b)  $f(x) = -3\sin x + \frac{4}{5}\cos x$

[8] A ball is propelled upward from a platform 144 feet high. Neglecting air resistance, its height  $s$  above the ground at time  $t$  is given by the position function  $s(t) = -16t^2 + 48t + 144$ , where  $t$  is seconds and  $s$  is in feet.

(a) Find the average velocity of the ball during the time interval  $[0, 4]$ .

(b) Find the instantaneous velocity when  $t = 3$ .

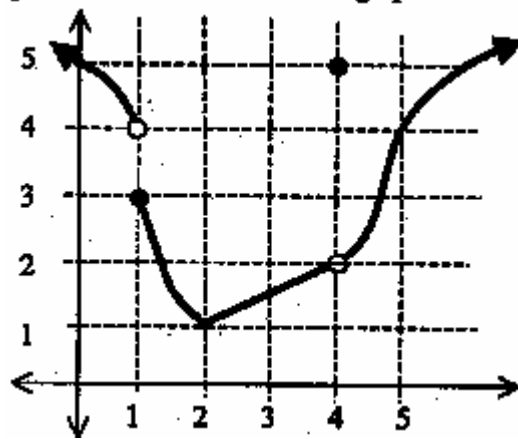
(c) Find the maximum height of the ball above the ground.

[9] Find an equation of the tangent line to the graph of  $f(x) = x^4 - 3x^3 + 5x^2 - 3x + 1$  at the point  $(2, 7)$ .

[10] Let  $f$  be the function whose graph is shown below. Use the graph to answer each of the following questions:

(a) Find  $\lim_{x \rightarrow 1^-} f(x)$

(b) Find  $\lim_{x \rightarrow 2} f(x)$



(c) Is  $f$  differentiable at  $x = 2$ ? Explain.

(d) Is  $f$  differentiable at  $x = 4$ ? Explain.