SI Session: March, 3rd & 5th, 2009
Tuesdays: 3:30 PM – 5:00 PM
Thursdays: 1:30 PM – 3:00 PM
& 3:30 PM – 5:00 PM
Room 1245 SNAD

Prof. Stockton : Calculus I
Spring 2009
SI Leader : Neil Jody

[1] Find the derivative of each function. DO NOT simplify your answer.

a) $f(x) = (x^2 - 2x)^{42} \sin x$

b) $g(x) = \sec^2 3x$

c) $h(x) = \dfrac{\sqrt{2x+3}}{x^3 + 2x - 1}$

d) $w(x) = \sin(\sec(\tan x))$

e) $f(x) = \dfrac{\tan x}{\sqrt{x^2 - 2x}}$

f) $g(x) = \sqrt{\cos(3x)}$

g) $h(x) = x^2 \csc(2x)$

h) $f(x) = \dfrac{\sqrt{2x+3}}{\sin(3x)}$

i) $f(x) = \left(\sec\left(x^2\right) + x\right)^{20}$

j) $f(x) = (x^2 + 3x)^{20} \tan x$

[2] Suppose $f'(9) = -2$ and $h(x) = x^2$. Calculate $(f \circ h)'(3)$.

[3] Find the following derivatives using *implicit differentiation*.

(a) $\dfrac{d}{dx}\left[x^2 y + 3xy^3 - x = 3\right]$

(b) $\dfrac{d}{dx}\left[x^2 = \dfrac{x+y}{x-y}\right]$

(c) $\dfrac{d}{dx}\left[\cos(xy^2) = y\right]$

[4] If $x = \cos y$, find $\dfrac{d^2y}{dx^2}$.

[5] For the curve given by $y^2 + x^2 y^3 + 11 = 4x$, find an equation of the tangent line at the point $(2,-1)$. Write the equation in the form $y = mx + b$.

[6] Find an equation of the line tangent to the graph of $y^3 - x^2 y + 4x = 7$ at the point $(-2,3)$.

[7] A runner starts at a point "A" and heads East at a rate of 10 ft/sec. One minute later another runner starts at "A" heading North at 8 ft/sec. At what rate is the distance between them changing 1 minute after the 2nd runner starts?

[8] Boyle's law for gases states that pv=c, for pressure p, volume v, and a constant c. At a certain instant the volume in $75\,in^3$, the pressure is $30\,lb/in^2$, and the pressure is decreasing at a rate of $2\,lb/in^2$ /min. At what rate is the volume changing at this instant?

[9] A point is moving along the graph of $y = x^3 - 3x^2$ in such a way that the *y*-coordinate is decreasing at the rate of 3 units/sec (i.e. $\frac{dy}{dt} = -3$). What is $\frac{dx}{dt}$ when $x = 1$?

[10] A man 6 ft tall is walking at a rate of 3 ft/s toward a streetlight 18 ft high. (a) At what rate is his shadow length changing? (b) How fast is the tip of his shadow moving?

Differentiation Properties and Rules:

1. $\frac{d}{dx}\left[c\,f(x)\right]=c\frac{d}{dx}\left[f(x)\right]$
2. $\frac{d}{dx}\left[f(x)\pm g(x)\right]=\frac{d}{dx}\left[f(x)\right]\pm\frac{d}{dx}\left[g(x)\right]$
3. $\frac{d}{dx}\left[f(x)g(x)\right]=f'(x)g(x)+f(x)g'(x)$
4. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{f'(x)g(x)-f(x)g'(x)}{\left(g(x)\right)^2}$
5. $\frac{d}{dx}\left[x^n\right]=nx^{n-1}$
6. $\frac{d}{dx}\left[f(g(x))\right]=f'(g(x))g'(x)$
7. $\frac{d}{dx}[\sin x]=\cos x$
8. $\frac{d}{dx}[\cos x]=-\sin x$
9. $\frac{d}{dx}[\tan x]=\sec^2 x$
10. $\frac{d}{dx}[\csc x]=-\csc x\cot x$
11. $\frac{d}{dx}[\sec x]=\sec x\tan x$
12. $\frac{d}{dx}[\cot x]=-\csc^2 x$

Trigonometric Identities (you <u>should</u> know):

1. $\sin^2(\theta)+\cos^2(\theta)=1$
2. $\tan^2(\theta)+1=\sec^2(\theta)$
3. $1+\cot^2(\theta)=\csc^2(\theta)$
4. $\sin(2\theta)=2\sin(\theta)\cos(\theta)$

$$\cos(2\theta)=\begin{cases}\cos^2(\theta)-\sin^2(\theta)\\ 2\cos^2(\theta)-1\\ 1-2\sin^2(\theta)\end{cases}$$

$$\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta$$

$$\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\sin\beta\cos\alpha$$

Properties of Exponents:

$\left(a^m\right)^n=a^{mn}$

$(ab)^n=a^n b^n$

$a^m\cdot a^n=a^{m+n}$

$\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}, b\neq 0$

$a^{-n}=\frac{1}{a^n}$

$\frac{a^m}{a^n}=a^{m-n}$

$a^0=1$

$a^1=a$

$a^{\frac{m}{n}}=\left(a^{\frac{1}{n}}\right)^m=\left(\sqrt[n]{a}\right)^m=\sqrt[n]{a^m}$

$\sqrt[n]{a}\cdot\sqrt[n]{b}=\sqrt[n]{a\cdot b}$

$\sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}, b\neq 0$

$b\sqrt[n]{a}\pm c\sqrt[n]{a}=(b\pm c)\sqrt[n]{a}$

Properties of Logarithms:

For all real numbers y, and all positive numbers a and x, where $a\neq 1$:

$\log_b x=y\Leftrightarrow b^y=x$

For $x>0$, $y>0$, $a>0$, $a\neq 1$, and any real number r:

$\log_b x^r=r\cdot\log_b x$

$\log_b xy=\log_b x+\log_b y$

$\log_b\left(\frac{x}{y}\right)=\log_b x-\log_b y$

For any positive real numbers x, a, and b, where $a\neq 1$ and $b\neq 1$:

$\log_b x=\frac{\log x}{\log b}=\frac{\ln x}{\ln b}=\frac{\log_a x}{\log_a b}$