

Directions: Evaluate and/or calculate as indicated.

1. Rolle's Theorem: Let f be continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b) . If $f(a)=f(b)$, then there is at least one point c in the interval (a,b) such that $f'(c)=0$.

What does this mean?

2. Mean-Value Theorem: Let f be continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b) . Then there is at least one point c in (a,b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

What does this mean?

3. Find the extrema of $f(x) = \sqrt[3]{x^2} - x$, on the interval $[-1, 2]$
4. A stone is dropped into a lake causing circular waves whose radii increase at a constant rate of 0.5m/sec. At what rate is the circumference of a wave changing when its radius is 4m?
5. Boyle's law for gases states that $pvc=c$, for pressure p , volume v , and a constant c . At a certain instant the volume is 75 in^3 , the pressure is 30 lb/in^2 , and the pressure is decreasing at a rate of $2 \text{ lb/in}^2/\text{min}$. At what rate is the volume changing at this instant?

6.
$$\frac{d}{dx} \left[x^2 = \frac{x+y}{x-y} \right]$$

7.
$$\frac{d}{dx} \left[\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{y}} = 1 \right]$$

8.
$$\frac{d}{dx} \left[\sqrt{x} \tan^3(\sqrt{x}) \right]$$

9.
$$\frac{d}{dx} \left[x^5 \sec\left(\frac{1}{x}\right) \right]$$

10.
$$\frac{d^2 y}{dx^2} \left[x \cos(5x) - \sin^2(x) \right]$$

11.
$$\frac{d^2 y}{dx^2} \left[x \tan\left(\frac{1}{x}\right) \right]$$

Tips:
$$\frac{d}{dx} [f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

12. If $y = \sec^3\left(\frac{\pi}{2} - x\right)$,

find equation of tangent line at $x = -\frac{\pi}{2}$

For the following find the derivative and critical values of f .

13. $f(x) = x - \sin(x)$

14. $f(x) = \frac{3x}{\sqrt{4x^2 + 1}}$

15. $f(x) = \sqrt{x^2 - 3x - 10}$

16. $f(x) = \frac{x^2 - 9}{(x+1)^2}$

17. $f(x) = \sqrt{x}(x-2)^2$

18. $f(x) = x - x^{2/3}$

19. $f(x) = \sin^2(x)$

20. $f(x) = \cos(x) - \sin(2x)$

21. $f(x) = \cos(2x) + \cos(x)$

22. $f(x) = \sin^2(x) + \cos(x)$

Find the Extrema and Mean Values on in the given intervals.

23. $f(x) = \cos(x); [\pi/2, 3\pi/2]$

24. $f(x) = \frac{1}{2}x - \sqrt{x}; [0, 4]$

25. $f(x) = \frac{1}{x^2} - \frac{4}{3x} + \frac{1}{3}; [1, 3]$

