SI Final Exam Review Session
December 7th & 8th 12:00 – 4:00 PM. Rm. 1223

Prof. Stockton : Calculus I : Fall 2007
SI Leader : Neil Jody

1. A snowball is melting in the hot Pasadena sun. At the moment when the radius of the snowball is 3 inches, the volume of the snowball is decreasing at 4π in^3/sec. How fast is the radius changing at that time?

2. A hot-air balloon rises vertically as a rope attached to the base of the balloon is released at a rate of 5ft/sec. The pulley that releases the rope is 20 feet from the spot on the ground directly below the balloon. At what rate is the balloon rising when 500 feet of rope have been let out?

3. A point is moving along the graph of $y = x^3 - 3x^2$ in such a way that the *y*-coordinate is decreasing at the rate of 3 units/sec (i.e. $\frac{dy}{dt} = -3$). What is $\frac{dx}{dt}$ when $x = 1$?

4. Sand falls from a conveyor belt onto a conical pile at a rate of $10\ \text{ft}^3/\text{min}$. The radius of the base is always equal to half of the pile's height. At what rate is the height of the pile increasing when the pile is 5 feet high?

5. Farmer John wants to create a pig pen. The pen is to be a rectangle divided into three sub-rectangles. He has exactly 300 feet of wire to create the pen. Find the dimensions of the pen with maximum area.

6. A box with a square base and no top is to have a volume of 108 cubic feet. Find the dimensions of the box which has the least surface area.

7. $\frac{d^2}{dx^2}\left[x\tan\left(\frac{1}{x}\right)\right]$

8. $\frac{d}{dx}\left[x^3 + y^3 = 3xy^2\right]$

9. $\lim_{x\to 0}\frac{\sin 5x}{x\cos 3x}$

10. $\lim_{x\to 4}\frac{x-4}{2-\sqrt{x}}$

11. $\lim_{x\to\infty}\frac{3x^2-5x}{4-2x^3}$

12. $\lim_{x\to\infty}\frac{\sin\sqrt{x}}{\sqrt{x}}$

13. $\lim_{x\to 0^+}\frac{-2\cos x}{\sin 5x}$

14. $\lim_{x\to 1^-} g(x)$ where $g(x) = \begin{cases} \frac{x}{x+2} & \text{if } x \le 1 \\ \frac{-2}{x-1} & \text{if } x > 1 \end{cases}$

For each of the following functions, find the open intervals on which the function is increasing or decreasing, and find the relative extrema.

41. $f(x) = x^{2/3}(16 - x^2)$

42. $g(x) = \frac{x^4+1}{x^2}$

43. $f(x) = \frac{12x}{x^2+4}$

For each of the following functions, find the intervals on which the function is concave up or down, and identify any inflection points.

44. $f(x) = -2x^4 + 8x^3$

45. $g(x) = 3x^5 + 10x^4 - 7$

For the following find the 1st and 2nd derivatives.

46. $f(x) = x - \sin(x)$

47. $f(x) = (1 - 2\sin x)^2$

48. $f(x) = \sin^2(x)$

49. $f(x) = \cos(x) - \sin(2x)$

50. $\int \cos x e^{\sin x} dx$

51. $\int_0^{\ln 3} \frac{e^{3x} - e^x}{e^{2x}} dx$

52. $\int \frac{e^{-x}}{1+e^{-x}} dx$

53. $\int x e^{x^2} dx$

Find $\frac{dy}{dx}$ in each case.

54. $y = \log_x 5$

55. $y = (\cos x)^x$

56. $y = e^x + x^e$

57. $y = x^2 e^{3x}$

58. $y = (\ln x)^3 + 3^{\ln x}$

15. $\lim_{x\to 1^-} \dfrac{3x-3}{|x-1|}$

16. $\lim_{x\to 2^-} \dfrac{x-4}{x^2-x-2}$

17. $\lim_{x\to -\infty} \cos\left(-\frac{3}{x^2}\right)$

18. $\lim_{x\to 0} \dfrac{\sin 4x}{\sin 3x}$

19. $\lim_{x\to -3} \dfrac{x^2+5x+6}{2x^2+5x-3}$

20. $\lim_{x\to -\infty} \dfrac{|x-2|}{2x+3}$

21. $\lim_{x\to 2^-} \dfrac{x+3}{x^2-6x+8}$

22. $\lim_{x\to 3^+} f(x)$ where $f(x)=\begin{cases} x^2-5 & \text{if } x\le 3 \\ 2x+3 & \text{if } x>3 \end{cases}$

23. $\lim_{x\to 1} \dfrac{\sqrt{x^2+3x}-2}{x-1}$

24. $\lim_{x\to \pi/2} \dfrac{\sin x}{x}$

25. $\lim_{x\to -1} \dfrac{2x^2+x-1}{x^2-2x-3}$

26. $\lim_{x\to 3} \dfrac{\sqrt{3x-5}-2}{x-3}$

27. $\lim_{x\to 3} \dfrac{\sin(x-3)}{x^2-9}$

28. $\lim_{x\to 0} \dfrac{\frac{1}{x+2}-\frac{1}{2}}{x}$

29. $\lim_{x\to 5^-} \dfrac{|x-5|}{x-5}$

30. $\lim_{x\to -2^+} \dfrac{x+3}{2x^2+7x+6}$

31. $\lim_{x\to 2^+} f(x)$ where $f(x)=\begin{cases} \dfrac{3}{x-1} & \text{if } x\le 2 \\ x^2+3x & \text{if } x>2 \end{cases}$

32. $\lim_{x\to -\infty} \dfrac{|x+2|}{2x+3}$

33. $\lim_{x\to \infty} \dfrac{5x^3+2x-1}{4-x^2}$

59. $y=\ln\left(\dfrac{x^2\sin x\sqrt{2x+3}}{(x^2+4)\ln x}\right)$

60. $y=\tan(\ln\sqrt{x})$

61. $y=\ln(\sin(3x))$

62. $\ln\left(xy^2\right)=\cos x$

63. $y=x^x$

Use the definition of derivative to find the derivative of each function.

64. $f(x)=\dfrac{x}{x-5}$

65. $h(x)=\dfrac{1}{\sqrt{x}}$

66. $g(x)=\sqrt{3x-2}$

67. $k(x)=\cos x$

Calculate $\dfrac{d}{dx}$ for the following

68. $g(x)=\sec^2 3x$

69. $h(x)=\dfrac{\sqrt{2x+3}}{x^3+2x-1}$

70. $w(x)=\sin(\sec(\tan x))$

71. $f(x)=\dfrac{\tan x}{\sqrt{x^2-2x}}$

72. $g(x)=\sqrt{\cos(3x)}$

73. $h(x)=x^2\csc(2x)$

74. $f(x)=\dfrac{\sqrt{2x+3}}{\sin(3x)}$

75. $f(x)=\sqrt{\cos(5x)}$

Calculate the following.

76. $\displaystyle\int \frac{x^3-x+2}{\sqrt{x}}dx$

77. $\displaystyle\int_3^6 x^2\sqrt{x-2}\ dx$

78. $\displaystyle\int \frac{\sin\left(\sqrt{x}+3\right)}{\sqrt{x}}\ dx$

79. $\displaystyle\int \frac{3\cos\left(4+\frac{1}{x}\right)}{x^2}dx$

80. $\displaystyle\int_0^{\pi/3} \sin x\cos^2 x\ dx$

34. $\int \frac{2x^2 - x + 3}{\sqrt{x}}\, dx$

35. $\int \left(\cos x + \sin x\right)^2 dx$

36. $\int \frac{3x + 6}{\sqrt[3]{x^2 + 4x - 3}} dx$

37. $\int \frac{x}{(4 - x)^3}\, dx$

38. $\int_0^{\pi/2} \sin x \sqrt{3\cos x + 1}\, dx$

39. $\int \sec^2\left(\frac{x}{3}\right) \tan^2\left(\frac{x}{3}\right)\, dx$

40. $\int_{-2}^{3} \left|2x - 4\right| dx$

$$\frac{Big\#}{Small\#} = Huge\# \Rightarrow \pm\infty$$

$$\frac{Small\#}{Big\#} = tiny\# \Rightarrow 0$$

$$\frac{d}{dx}\left[\mathrm{f}(x)g(x)\right] = \mathrm{f}(x)g'(x) + g(x)\mathrm{f}'(x)$$

$$\frac{d}{dx}\left[\frac{\mathrm{f}(x)}{g(x)}\right] = \frac{g(x)\mathrm{f}'(\mathrm{x}) - \mathrm{f}(\mathrm{x})g'(x)}{\left(g(x)\right)^2}$$

81. $\int_1^e \frac{(\ln x)^4}{x} dx$

82. $\int_4^9 \frac{1}{\sqrt{x}\left(2\sqrt{x} - 3\right)} dx$

83. $\int \frac{3x}{4 + x^2} dx$

84. $\int \frac{\sin(\ln x)}{x} dx$

85. $\int_0^{\pi/6} \frac{\cos x}{2\sin x + 1} dx$

86. $\int \frac{\tan\sqrt{x}}{\sqrt{x}} dx$

$$\int cf(x)\, dx = c\int f(x)\, dx$$

$$\int \left[f(x) \pm g(x)\right] dx = \int f(x)\, dx \pm \int g(x)\, dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \text{ if } n \neq -1$$

$$\int x^n dx = \ln\left|x\right| + C, \text{if } n = -1$$

$$\int b^x dx = \frac{b^x}{\ln b} + C, \left(0 < b, b \neq 1\right)$$

$$\int \sin x\, dx = -\cos x + C$$

$$\int \cos x\, dx = \sin x + C$$

$$\int \sec^2 x\, dx = \tan x + C$$

$$\int \csc^2 x\, dx = -\cot x + C$$

$$\int \sec x \tan x\, dx = \sec x + C$$

$$\int \csc x \cot x\, dx = -\csc x + C$$

$$\int \sec x\, dx = \ln\left|\sec x + \tan x\right| + C$$

$$\int \tan x\, dx = -\ln\left|\cos x\right| + C$$

$$\int \cot x\, dx = \ln\left|\sin x\right| + C$$

$$\int \csc x\, dx = \ln\left|\csc x - \cot x\right| + C$$